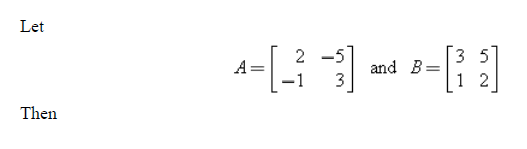
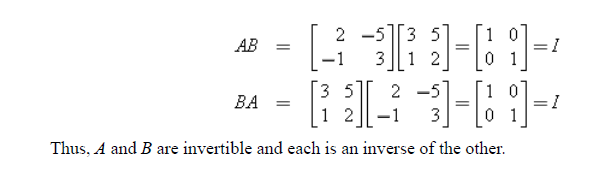
**1.5 Inverse of a Matrix**

**Definition:** If A is a square matrix, and if a matrix B of the same size can be found such that

then A is said to be **invertible** (or **nonsingular**) and B is called an **inverse** of A. If no such matrix B can be found, then A is said to be **singular**.

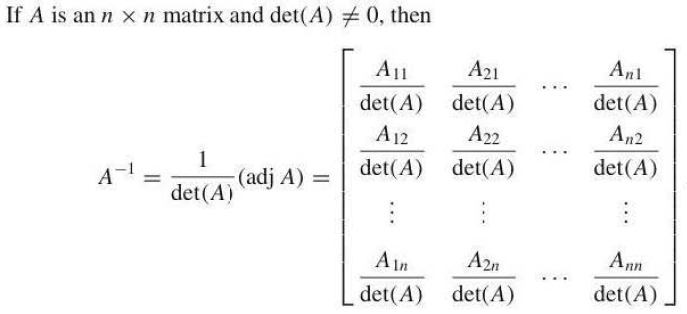
The relationship is not changed by interchanging A and B, so if A is invertible and B is an inverse of A, then it is also true that B is invertible, and A is an inverse of B. Thus, when ,we say that A and B are inverses of one another.

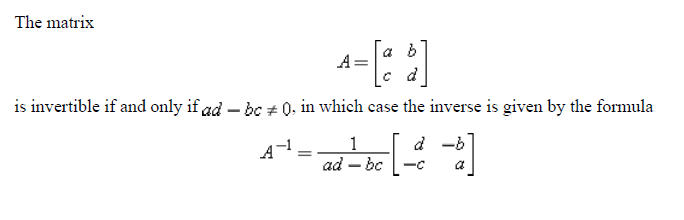


**Remarks:**

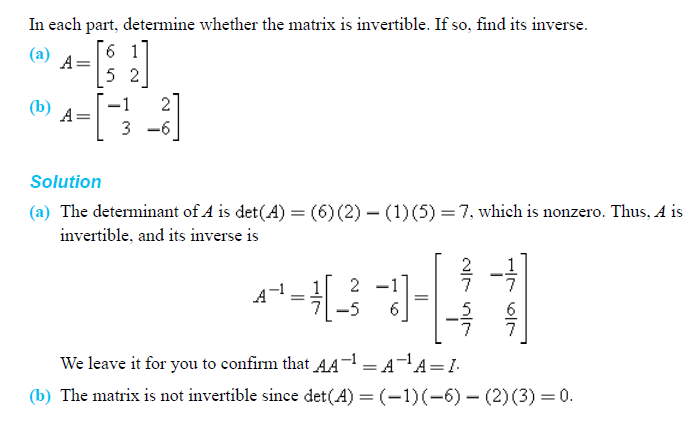
1. *In general, a square matrix with a row or column of zeros is singular.*
2. *An invertible matrix has exactly one inverse.*

**Formula for the Inverse of a matrix:**

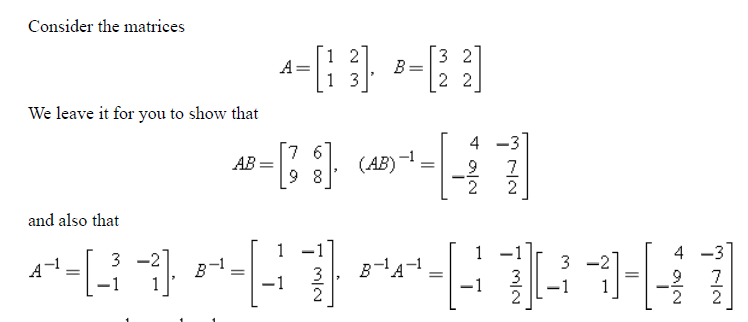
****

For a 2x2 matrix this formula is easy to use. For example 

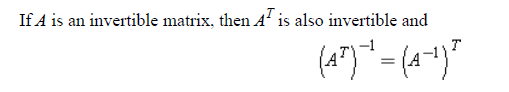
Example 1:



Example 2:



**Remark:**

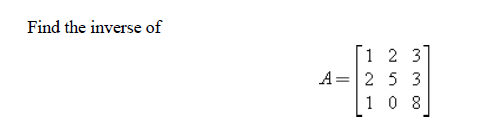


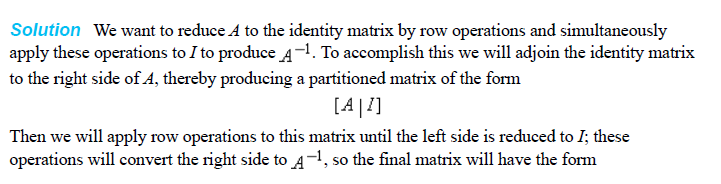
For a matrix of order higher than 2x2 we use the inversion algorithm to find its inverse (based on steps used for the reduced row echelon form).

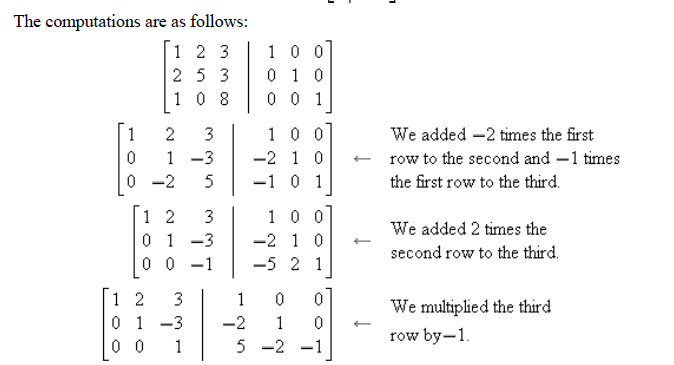
**Inversion Algorithm**

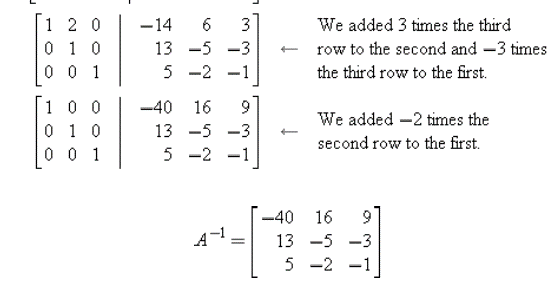
To find the inverse of an invertible matrix *A*, find a sequence of elementary row operations that reduces *A* to the identity and then perform that same sequence of operations on to obtain .

Example 3:



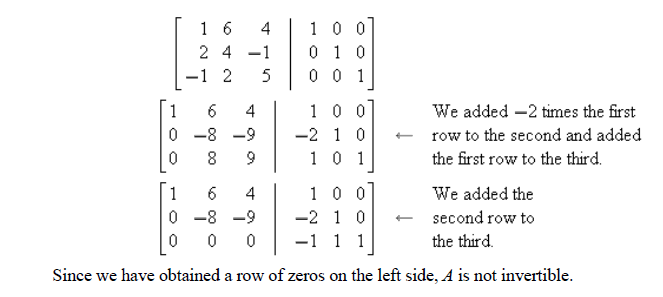






Example 4:





**Work to do**

**Q1.** Determine whether the statement is true or false, and justify your answer.

1. For all square matrices *A* and *B* of the same size

(ii) The product of two elementary matrices of the same size must be an elementary matrix.

(iii) If *A* is an matrix that is not invertible, then the linear system has infinitely many solutions.

(iv) It is impossible for a linear system of linear equations to have exactly two solutions.

(v) If *A* and *B* are invertible matrices of the same size, then *AB* is invertible and

(vi) Every elementary matrix is invertible.

(vii) If *A* is invertible and a multiple of the first row of *A* is added to the second row, then the resulting matrix is invertible.

(viii) If the linear system has a unique solution, then the linear system also must have a unique solution.

**Q2.** Use the inversion algorithm to find the inverses of the given matrices, if exist.



**Q3.** Find all values of *c*, if any, for which the given matrix is invertible.